

NCATE/NCTM Program Standards (2003) Programs for Initial Preparation of Mathematics Teachers

Standards for Secondary Mathematics Teachers

Process Standards (Standards 1-7)

The process standards are based on the belief that mathematics must be approached as a unified whole. Its concepts, procedures, and intellectual processes are so interrelated that, in a significant sense, its “whole is greater than the sum of the parts.” This approach would best be addressed by involvement of the mathematics content, mathematics education, education, and field experience faculty working together in developing the candidates’ experiences.

Likewise, the response to the disposition standard will require total faculty input. This standard addresses the candidates’ nature and temperament relative to being a mathematician, an instructor, a facilitator of learning, a planner of lessons, a member of a professional community, and a communicator with learners and their families.

Standard 1: Knowledge of Mathematical Problem Solving

Candidates know, understand, and apply the process of mathematical problem solving.

Indicators

- 1.1 Apply and adapt a variety of appropriate strategies to solve problems.
- 1.2 Solve problems that arise in mathematics and those involving mathematics in other contexts.
- 1.3 Build new mathematical knowledge through problem solving.
- 1.4 Monitor and reflect on the process of mathematical problem solving.

Standard 2: Knowledge of Reasoning and Proof

Candidates reason, construct, and evaluate mathematical arguments and develop an appreciation for mathematical rigor and inquiry.

Indicators

- 2.1 Recognize reasoning and proof as fundamental aspects of mathematics.
- 2.2 Make and investigate mathematical conjectures.
- 2.3 Develop and evaluate mathematical arguments and proofs.
- 2.4 Select and use various types of reasoning and methods of proof.

Standard 3: Knowledge of Mathematical Communication

Candidates communicate their mathematical thinking orally and in writing to peers, faculty, and others.

Indicators

- 3.1 Communicate their mathematical thinking coherently and clearly to peers, faculty, and others.
- 3.2 Use the language of mathematics to express ideas precisely.
- 3.3 Organize mathematical thinking through communication.
- 3.4 Analyze and evaluate the mathematical thinking and strategies of others.

Standard 4: Knowledge of Mathematical Connections

Candidates recognize, use, and make connections between and among mathematical ideas and in contexts outside mathematics to build mathematical understanding.

Indicators

- 4.1 Recognize and use connections among mathematical ideas.
- 4.2 Recognize and apply mathematics in contexts outside of mathematics.
- 4.3 Demonstrate how mathematical ideas interconnect and build on one another to produce a coherent whole.

Standard 5: Knowledge of Mathematical Representation

Candidates use varied representations of mathematical ideas to support and deepen students' mathematical understanding.

Indicators

- 5.1 Use representations to model and interpret physical, social, and mathematical phenomena.
- 5.2 Create and use representations to organize, record, and communicate mathematical ideas.
- 5.3 Select, apply, and translate among mathematical representations to solve problems.

Standard 6: Knowledge of Technology

Candidates embrace technology as an essential tool for teaching and learning mathematics.

Indicator

- 6.1 Use knowledge of mathematics to select and use appropriate technological tools, such as but not limited to, spreadsheets, dynamic graphing tools, computer algebra systems, dynamic statistical packages, graphing calculators, data-collection devices, and presentation software.

Standard 7: Dispositions

Candidates support a positive disposition toward mathematical processes and mathematical learning.

Indicators

- 7.1 Attention to equity
- 7.2 Use of stimulating curricula
- 7.3 Effective teaching
- 7.4 Commitment to learning with understanding
- 7.5 Use of various assessments
- 7.6 Use of various teaching tools including technology

Pedagogy (Standard 8)

In addition to knowing students as learners, mathematics teacher candidates should develop knowledge of and ability to use and evaluate instructional strategies and classroom organizational models, ways to represent mathematical concepts and procedures, instructional materials and resources, ways to promote discourse, and means of assessing student understanding. This section on pedagogy is to address this knowledge and skill.

Standard 8: Knowledge of Mathematics Pedagogy

Candidates possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning.

Indicators

- 8.1 Selects, uses, and determines suitability of the wide variety of available mathematics curricula and teaching materials for all students including those with special needs such as the gifted, challenged and speakers of other languages.
- 8.2 Selects and uses appropriate concrete materials for learning mathematics.
- 8.3 Uses multiple strategies, including listening to and understanding the ways students think about mathematics, to assess students' mathematical knowledge.
- 8.4 Plans lessons, units and courses that address appropriate learning goals, including those that address local, state, and national mathematics standards and legislative mandates.
- 8.5 Participates in professional mathematics organizations and uses their print and on-line resources.
- 8.6 Demonstrates knowledge of research results in the teaching and learning of mathematics.
- 8.7 Uses knowledge of different types of instructional strategies in planning mathematics lessons.
- 8.8 Demonstrates the ability to lead classes in mathematical problem solving and in developing in-depth conceptual understanding, and to help students develop and test generalizations.

8.9 Develop lessons that use technology's potential for building understanding of mathematical concepts and developing important mathematical ideas.

Content (Standards 9-15)

Candidates' comfort with, and confidence in, their knowledge of mathematics affects both what they teach and how they teach it. Knowing mathematics includes understanding specific concepts and procedures as well as the process of doing mathematics. That knowledge is the subject of the following standards.

Standard 9: Knowledge of Number and Operation

Candidates demonstrate computational proficiency, including a conceptual understanding of numbers, ways of representing number, relationships among number and number systems, and meanings of operations.

Indicators

- 9.1 Analyze and explain the mathematics that underlies the procedures used for operations involving integers, rational, real, and complex numbers.
- 9.2 Use properties involving number and operations, mental computation, and computational estimation.
- 9.3 Provide equivalent representations of fractions, decimals, and percents.
- 9.4 Create, solve, and apply proportions.
- 9.5 Apply the fundamental ideas of number theory.
- 9.6 Make sense of large and small numbers and use scientific notation.
- 9.7 Compare and contrast properties of numbers and number systems.
- 9.8 Represent, use, and apply complex numbers.
- 9.9 Recognize matrices and vectors as systems that have some of the properties of the real number system.
- 9.10 Demonstrate knowledge of the historical development of number and number systems including contributions from diverse cultures.

Standard 10: Knowledge of Different Perspectives on Algebra

Candidates emphasize relationships among quantities including functions, ways of representing mathematical relationships, and the analysis of change.

Indicators

- 10.1 Analyze patterns, relations, and functions of one and two variables.
- 10.2 Apply fundamental ideas of linear algebra.
- 10.3 Apply the major concepts of abstract algebra to justify algebraic operations and formally analyze algebraic structures.
- 10.4 Use mathematical models to represent and understand quantitative relationships.

10.5 Use technological tools to explore algebraic ideas and representations of information and in solving problems.

10.6 Demonstrate knowledge of the historical development of algebra including contributions from diverse cultures.

Standard 11: Knowledge of Geometries

Candidates use spatial visualization and geometric modeling to explore and analyze geometric shapes, structures, and their properties.

Indicators

11.1 Demonstrate knowledge of core concepts and principles of Euclidean and non-Euclidean geometries in two and three dimensions from both formal and informal perspectives.

11.2 Exhibit knowledge of the role of axiomatic systems and proofs in geometry.

11.3 Analyze characteristics and relationships of geometric shapes and structures.

11.4 Build and manipulate representations of two- and three- dimensional objects and visualize objects from different perspectives.

11.5 Specify locations and describe spatial relationships using coordinate geometry, vectors, and other representational systems.

11.6 Apply transformations and use symmetry, similarity, and congruence to analyze mathematical situations.

11.7 Use concrete models, drawings, and dynamic geometric software to explore geometric ideas and their applications in real-world contexts.

11.8 Demonstrate knowledge of the historical development of Euclidean and non-Euclidean geometries including contributions from diverse cultures.

Standard 12: Knowledge of Calculus

Candidates demonstrate a conceptual understanding of limit, continuity, differentiation, and integration and a thorough background in the techniques and application of the calculus.

Indicators

12.1 Demonstrate a conceptual understanding of and procedural facility with basic calculus concepts.

12.2 Apply concepts of function, geometry, and trigonometry in solving problems involving calculus.

12.3 Use the concepts of calculus and mathematical modeling to represent and solve problems taken from real-world contexts.

12.4 Use technological tools to explore and represent fundamental concepts of calculus.

12.5 Demonstrate knowledge of the historical development of calculus including contributions from diverse cultures.

Standard 13: Knowledge of Discrete Mathematics

Candidates apply the fundamental ideas of discrete mathematics in the formulation and solution of problems.

Indicators

- 13.1 Demonstrate knowledge of basic elements of discrete mathematics such as graph theory, recurrence relations, finite difference approaches, linear programming, and combinatorics.
- 13.2 Apply the fundamental ideas of discrete mathematics in the formulation and solution of problems arising from real-world situations.
- 13.3 Use technological tools to solve problems involving the use of discrete structures and the application of algorithms.
- 13.4 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

Standard 14: Knowledge of Data Analysis, Statistics, and Probability

Candidates demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability.

Indicators

- 14.1 Design investigations, collect data, and use a variety of ways to display data and interpret data representations that may include bivariate data, conditional probability and geometric probability.
- 14.2 Use appropriate methods such as random sampling or random assignment of treatments to estimate population characteristics, test conjectured relationships among variables, and analyze data.
- 14.3 Use appropriate statistical methods and technological tools to describe shape and analyze spread and center.
- 14.4 Use statistical inference to draw conclusions from data.
- 14.5 Identify misuses of statistics and invalid conclusions from probability.
- 14.6 Draw conclusions involving uncertainty by using hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and conclusions.
- 14.7 Determine and interpret confidence intervals.
- 14.8 Demonstrate knowledge of the historical development of statistics and probability including contributions from diverse cultures.

Standard 15: Knowledge of Measurement

Candidates apply and use measurement concepts and tools.

Indicators

- 15.1 Recognize the common representations and uses of measurement and choose tools and units for measuring.

15.2 Apply appropriate techniques, tools, and formulas to determine measurements and their application in a variety of contexts.

15.3 Completes error analysis through determining the reliability of the numbers obtained from measures.

15.4 Demonstrate knowledge of the historical development of measurement and measurement systems including contributions from diverse cultures.

Field-Based Experiences (Standard 16)

The development of mathematics teacher candidates should include opportunities to examine the nature of mathematics, how it should be taught and how students learn mathematics; observe and analyze a range of approaches to mathematics teaching and learning, focusing on the tasks, discourse, environment and assessment; and work with a diverse range of students individually, in small groups, and in large class settings.

Standard 16: Field-Based Experiences

Candidates complete field-based experiences in mathematics classrooms.

Indicators

16.1 Engage in a sequence of planned opportunities prior to student teaching that includes observing and participating in both middle and secondary mathematics classrooms under the supervision of experienced and highly qualified teachers.

16.2 Experience full-time student teaching in secondary mathematics that is supervised by a highly qualified teacher and a university or college supervisor with secondary mathematics teaching experience.

16.3 Demonstrate the ability to increase students' knowledge of mathematics.